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# Conformal gauge fixing and Faddeev–Popov determinant in 2-dimensional Regge gravity<sup>1</sup>

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## Abstract

By regularizing the conical singularities by means of a segment of a sphere or pseudosphere and then taking the regulator to zero, we compute exactly the Faddeev–Popov determinant related to the conformal gauge fixing for a piece-wise flat surface with the topology of the sphere. The result is analytic in the opening angles of the conical singularities in the interval  $(\pi, 4\pi)$  and in the smooth limit goes over to the continuum expression. The Riemann-Roch relation on the dimensions of  $\ker(L^\dagger L)$  and  $\ker(LL^\dagger)$  is satisfied.

At present we have a well developed and consistent theory of two dimensional gravity in the continuum formulation [1, 2] accompanied by a collection of exact results [3].

In order to extend the predictive range of such a theory, discretized versions have been put forward, which can be subject to numerical simulations. In practice two methods have been proposed and exploited: the first is the Regge approach [4, 5, 6] and the second the so called dynamically triangulated random surfaces approach [7] which in two dimensions has strong connections with the matrix models [8].

Here we shall be concerned with the Regge approach which was historically the first. Since the beginning the main discussion with regard to the quantum formulation of Regge gravity centered about the integration measure to be adopted in the functional integral and the role played by the diffeomorphisms [6, 9, 10].

Here we shall adhere to the viewpoint that the only difference between Regge gravity and the usual formulation of quantum gravity, is that while in the continuum one integrates (or try to integrate) over all surfaces, in the Regge approach one limits oneself to integrate only on the piecewise flat surfaces. When the number of faces goes to infinity one hopes to recover the continuum theory.

From this viewpoint there is no difference in the definition of diffeomorphism between the continuum and the Regge formulation [6, 10, 11]. A strictly related point is that of the integration measure. The usual procedure which has proven successful in gauge theories is to start from the integration over to the local gauge variables; if the gauge volume turns out to be infinite, a gauge fixing has to be introduced and the corresponding Faddeev-Popov determinant computed. The last step is avoided in the lattice formulation of QCD as the local integration variables are taken as the elements of the gauge group which is compact and possesses a finite volume.

In gravity we shall stick to integrating over the analogous of the gauge variables i.e. the metric. In order to provide a measure, a distance functional has to be introduced and we shall adopt the De-Witt metric

$$(\delta g^{(1)}, \delta g^{(2)}) = \int \sqrt{g} \, \delta g_{\mu\nu}^{(1)} \left( g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha} + C g^{\mu\nu} g^{\alpha\beta} \right) \delta g_{\alpha\beta}^{(2)}, \quad (1)$$

which is the only ultra-local metric invariant under diffeomorphisms.

Due to the infinite volume of the diffeomorphisms a gauge fixing has to be introduced and the most suitable one in two dimensions appears to be the conformal

gauge fixing  $g_{\mu\nu} = e^{2\sigma} \hat{g}_{\mu\nu}(\tau_i)$ , where  $\tau_i$  are the Teichmüller parameters. In a classical series of papers [1, 2] the following expression was reached for the partition function of two dimensional euclidean quantum gravity

$$\mathcal{Z} = \int \mathcal{D}[\sigma] d\tau_i \sqrt{\frac{\det'(L^\dagger L)}{\det(\Psi_k, \Psi_l) \det(\Phi_a, \Phi_b)}} \quad (2)$$

where

$$\begin{aligned} (L\xi)_{\mu\nu} &= \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu - g_{\mu\nu} \nabla^\rho \xi_\rho \\ (L^\dagger h)_\nu &= -4 \nabla^\mu h_{\mu\nu} \end{aligned} \quad (3)$$

being  $\xi_\mu$  a vector field and  $h_{\mu\nu}$  a symmetric traceless tensor field.  $\mathcal{D}[\sigma]$  is the functional integration measure induced by the distance

$$(\delta\sigma^{(1)}, \delta\sigma^{(2)}) = \int \sqrt{\hat{g}} e^{2\sigma} \delta\sigma^{(1)} \delta\sigma^{(2)}. \quad (4)$$

$\Psi_k$  and  $\Phi_a$  are respectively the zero modes of  $L$  and  $L^\dagger$ . For the sphere topology, to which we shall refer from now on, there are no Teichmüller parameters and hence no zero modes of  $L^\dagger$ . The dependence on  $\sigma$  of the integrand in (2) can be factorized in the expression  $e^{-26S_L}$  where

$$S_L[\sigma, \hat{g}(\tau_i)] = \frac{1}{24\pi} \int d^2x \sqrt{\hat{g}} [\hat{g}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + R_{\hat{g}} \sigma] \quad (5)$$

$e^{-26S_L}$  is the (square-root) of the Faddeev-Popov determinant related to the conformal gauge fixing.

A Regge surface whose singularities have location  $\omega_i$  in the projective plane and angular aperture  $2\pi\alpha_i$  ( $\alpha_i = 1$  is the plane), is described by a conformal factor [12]  $e^{2\sigma} = e^{2\lambda_0} \prod_i |\omega - \omega_i|^{2(\alpha_i - 1)}$  which in the neighborhood of  $\omega_i$  becomes  $e^{2\lambda_i} |\omega - \omega_i|^{2(\alpha_i - 1)}$  with  $e^{2\lambda_i} = e^{2\lambda_0} \prod_{j \neq i} |\omega_i - \omega_j|^{2(\alpha_j - 1)}$ . In the conformal gauge  $L$  and  $L^\dagger$  assume the form

$$L = e^{2\sigma} \frac{\partial}{\partial \bar{\omega}} e^{-2\sigma}, \quad L^\dagger = -e^{-2\sigma} \frac{\partial}{\partial \omega}. \quad (6)$$

If now we try to evaluate  $S_L$  for a conformal factor describing a piecewise flat geometry we obtain a divergent result. Nevertheless  $\frac{\det'(L^\dagger L)}{\det(\Psi_k, \Psi_l)}$  can be defined also for a Regge surface by means of the  $Z$ -function regularization [13] which gives

$$-\ln(\det'(L^\dagger L)) = Z'(s)|_{s=0} = \gamma_E Z(0) + \text{Finite}_{\epsilon \rightarrow 0} \int_\epsilon^\infty \frac{dt}{t} \text{Tr}'(e^{-tL^\dagger L}) \quad (7)$$

where  $\det'$  and  $\text{Tr}'$  mean that the zero modes are excluded. Following the standard procedure developed in the continuum approach,  $Z'(0)$  will be computed by first

performing a variation  $\delta\sigma$  in the conformal factor and later integrating back the result. For the case at hand we have

$$\delta Z'(0) = \gamma_E \delta c_0^K + \text{Finite}_{\epsilon \rightarrow 0} \int d^2x [4\delta\sigma(x)(K(x, x, \epsilon) - \sum_k |\Psi_k(x)|^2) + \\ - 2\delta\sigma(x)H(x, x, \epsilon)] \quad (8)$$

where  $K$  is the heat kernel of the operator  $L^\dagger L$ ,  $H$  is the heat kernel of the operator  $LL^\dagger$  and  $c_0^K = Z(0) + \dim \ker (L^\dagger L)$  is the constant term in the asymptotic expansion of the trace of  $K(x, x', t)$ .

Aurell and Salomonson [14] gave the determinant of the scalar Laplace-Beltrami operator for a piece-wise flat surface with the topology of the sphere and for also for some compact domains of the plane.

In our case [15] the main point is the computation of  $K(x, x', t)$  and  $H(x, x', t)$  which respect the correct boundary conditions imposed by the nature of the vector field  $\xi$  and of the tensor field  $h$ . In the neighborhood of  $\omega_i$  the conical singularity cone can be described in the  $z = x + iy$  plane, by a wedge of angular opening  $2\pi\alpha = 2\pi\alpha_i$ . The conformal variation considered here is the same adopted by [14] for the scalar case, and it takes from a cone with some opening angle  $2\pi\alpha$  and scale factor  $\lambda = \lambda_i$  to a cone with varied opening angle  $\alpha + \delta\alpha$  and scale factor  $\lambda + \delta\lambda$ . Such a conformal transformation is described by the variation in the  $z$ -plane

$$\delta\sigma(z, \bar{z}) = (\delta\lambda - \lambda \frac{\delta\alpha}{\alpha}) + \frac{\delta\alpha}{\alpha} \log(\alpha|z|). \quad (9)$$

The spectral representation of the heat kernel of  $L^\dagger L$  on the cone is given by

$$K_{\alpha, \delta}(x, x'; t) = \frac{1}{2\pi\alpha} \left\{ \sum_{n=0}^{\infty} e^{i(\phi - \phi') \frac{n+\delta}{\alpha}} \int_0^{\infty} J_{\frac{n+\delta}{\alpha}}(r\mu) J_{\frac{n+\delta}{\alpha}}(r'\mu) e^{-t\mu^2} \mu d\mu + \right. \\ \left. + \sum_{n=1}^{\infty} e^{-i(\phi - \phi') \frac{n-\delta}{\alpha}} \int_0^{\infty} J_{\frac{n-\delta}{\alpha}}(r\mu) J_{\frac{n-\delta}{\alpha}}(r'\mu) e^{-t\mu^2} \mu d\mu \right\} \quad (10)$$

where  $\delta = \alpha \bmod 1$  and is valid for  $\text{Re } \nu > -1$  ( $\nu$  is the index of the Bessel function). For the heat kernel of the operator  $LL^\dagger$  the same representation holds with  $\delta = 2\alpha \bmod 1$ .

One can compute the constant term  $c_0^K$  in the asymptotic expansion of the heat kernel (10) by a well know procedure [16] to obtain for  $K$

$$c_0^K = \frac{\delta(\delta - 1)}{2\alpha} + \frac{1 - \alpha^2}{12\alpha}. \quad (11)$$

On the other hand the self adjoint extensions of  $L^\dagger L$  and  $LL^\dagger$  depend on the boundary conditions one imposes on the eigenfunction at the singularities. The choice of Dowker and of Aurell and Salomonson for the Laplace-Beltrami operator is Dirichlet boundary conditions. This is equivalent to imposing for the phase-shift

$\delta$  in eq.(10) the restriction  $0 < \delta \leq 1$ . This gives for small angular deficits i.e.  $\alpha = 1 + \varepsilon$ , for  $\varepsilon < 0$ ,  $c_0^K = \frac{\varepsilon}{3} + O(\varepsilon^2)$  and  $c_0^H = \frac{5\varepsilon}{6} + O(\varepsilon^2)$  while for  $\varepsilon > 0$ ,  $c_0^K = -\frac{2\varepsilon}{3} + O(\varepsilon^2)$  and  $c_0^H = -\frac{7}{6}\varepsilon + O(\varepsilon^2)$ .

Such a result is non analytic in  $\alpha$  near the flat space and it gives the wrong continuum limit ( $\varepsilon \rightarrow 0$ ) for the FP determinant, both for positive and negative  $\varepsilon$ .

The reason of such a failure can be understood as follows: Dirichlet boundary conditions are equivalent to cutting off the tip of the cone; on the other hand the meaning of the tip of the cone of the Regge surface is that of a locus of infinite curvature. Thus we looked at the problem of treating the cone as the limit case of a regular geometry: for positive curvature we described the tip of the cone as a segment of a sphere which connects smoothly with the cone and for negative curvature we described the tip of the cone as a segment of the Poincaré pseudo-sphere of constant negative curvature.

The limit we are interested in, is the one of the radius of the sphere going to zero, keeping constant the integrated curvature. The sphere of radius  $\frac{1}{2}\rho$ , of constant curvature  $R = -2e^{-2\sigma}\Delta\sigma = 8\rho^{-2}$  or the pseudo-sphere of constant curvature  $R = -8\rho^{-2}$  are described on the complex  $\omega$  plane by the conformal factor  $e^{2\sigma} = (1 \pm u\bar{u})^{-2}$  with  $u = \omega/\rho$ . In order to proceed one needs to solve the eigenvalue problem on such regularized cones.

Solving explicitly the eigenvalue equation we find, for the eigensolutions with orbital angular momentum  $m$  on the sphere

$$m = n \geq 0 \quad \xi^{(n)} = \frac{u^n}{(1 + u\bar{u})^2} {}_2F_1(\gamma_1 + 2, 1 - \gamma_1; n + 1; \frac{u\bar{u}}{1 + u\bar{u}}) \quad (12)$$

$$m = -n \leq 0 \quad \xi^{(n)} = \bar{u}^n {}_2F_1(\gamma_1, -1 - \gamma_1; n + 1; \frac{u\bar{u}}{1 + u\bar{u}}) \quad (13)$$

where  $\gamma_1 = \frac{1}{2}(-1 + \sqrt{9 + 4(\rho\mu)^2})$ .

Similar solutions are found in the case of the pseudosphere. Such solutions have to be matched to the exterior solution on the cone by imposing the continuity of the logarithmic derivative of  $e^{-2\sigma}\xi$  with respect to  $\bar{\omega}$  at  $|\omega| = \tau_0$  (being  $\tau_0$  the radius on the  $\omega$ -plane at which the sphere connects to the cone) as required by the structure of eigenvalue equation  $e^{-2\sigma} \frac{\partial}{\partial \omega} e^{2\sigma} \frac{\partial}{\partial \bar{\omega}} e^{-2\sigma} \xi = -\mu^2 \xi$ . The general eigensolution on the exterior cone for orbital angular momentum  $m$  has the form

$$\xi_{\text{ext}}^{(m)} = \left(\frac{u}{\bar{u}}\right)^{\frac{m}{2}} (u\bar{u})^{\frac{\alpha-1}{2}} \left[ a(\rho) J_\gamma(2\rho\mu p(u\bar{u})^{\frac{\alpha}{2}}) + b(\rho) J_{-\gamma}(2\rho\mu p(u\bar{u})^{\frac{\alpha}{2}}) \right] \quad (14)$$

where  $\gamma = \frac{m+\alpha-1}{\alpha}$  and  $p = \frac{(u_0\bar{u}_0)^{\frac{1-\alpha}{2}}}{\alpha(1 \pm u_0\bar{u}_0)}$ . The coefficients  $a(\rho)$  and  $b(\rho)$  are fixed by imposing the matching conditions at  $\tau_0$ . Letting  $\rho \rightarrow 0$  (and thus  $\tau_0 \rightarrow 0$ ) gives the result [15] that for the opening of the cone  $2\pi\alpha$  with  $\frac{1}{2} < \alpha < 2$ , only the term  $J_{\frac{m+\alpha-1}{\alpha}}$  survives for  $m \geq 0$ , while for  $m < 0$  the surviving term is  $J_{-\frac{m+\alpha-1}{\alpha}}$ .

Going over to the coordinate  $z$ , the heat kernel  $K(x, x'; t)$  is thus given by (10) with  $\delta = \alpha - 1$ .

We come now to the heat kernel  $H$  for the field  $h$ . The requirement [2] that  $\det'(L^\dagger L) = \det'(LL^\dagger)$  fixes the eigenfunctions of  $LL^\dagger$  to  $h = L\xi$ . Thus the eigenfunctions of  $LL^\dagger$  are given in the  $z$  coordinate for  $m \geq 0$  by

$$\frac{\partial}{\partial \bar{z}} \left[ \left( \frac{z}{\bar{z}} \right)^{\frac{\gamma}{2}} J_\gamma(2\mu(z\bar{z})^{\frac{1}{2}}) \right] \quad (15)$$

which through a well known identity on the Bessel functions equals

$$- \mu \left( \frac{z}{\bar{z}} \right)^{\frac{\gamma+1}{2}} J_{\gamma+1}(2\mu(z\bar{z})^{\frac{1}{2}}), \quad (16)$$

while for  $m < 0$  they are given by

$$\mu \left( \frac{z}{\bar{z}} \right)^{\frac{\gamma+1}{2}} J_{-\gamma-1}(2\mu(z\bar{z})^{\frac{1}{2}}), \quad (17)$$

always with  $\gamma = \frac{m + \alpha - 1}{\alpha}$ . The net result is that the heat kernel  $H$  is given by (10) with  $\delta = 2\alpha - 1$ .

Applying Dowker's procedure to  $K = K_{\alpha, \alpha-1}$ , eq.(10), we obtain

$$c_0^K = \frac{(\alpha - 1)(\alpha - 2)}{2\alpha} + \frac{1 - \alpha^2}{12\alpha} \quad (18)$$

and for  $H = K_{\alpha, 2\alpha-1}$

$$c_0^H = \frac{(2\alpha - 1)(2\alpha - 2)}{2\alpha} + \frac{1 - \alpha^2}{12\alpha}. \quad (19)$$

We notice that the  $c_0$  are analytic in  $\alpha_i$  and  $2(c_0^K - c_0^H) = 3(1 - \alpha_i)$ . This holds for the contribution to  $Z(0) + \dim \ker L^\dagger L$  of a single conical singularity. Thus for a generic compact surface without boundary due to the local nature of the coefficients of the asymptotic expansion of the trace of the heat kernel [16] such a relation becomes  $2(c_0^K - c_0^H) = 3 \sum_i (1 - \alpha_i) = 3\chi$  where the sum runs over the vertices and  $\chi$  is the Euler characteristic of the surface, in agreement with the Riemann-Roch index theorem applied to  $L^\dagger L$  and  $LL^\dagger$  [2]. This provides an interesting check of the consistency of our regularization procedure.

We remark that the obtained results on the behavior of the eigenfunctions at the origin in the limit when the regulator goes to zero are largely independent of the details of the regularization of the tip of the cone. In fact the effect of our regularization is that to impose (apart from a correction that behaves like  $\rho^2$  and that vanishes in the limit  $\rho \rightarrow 0$ ) a fixed logarithmic derivative of  $e^{-2\sigma}\xi$  at the boundary, combined with the fact that for  $m \geq 0$  the regular eigenfunction of

$L^\dagger L$  to the null eigenvalue has the form  $e^{2\sigma}\omega^m$ . The advantage of the spherical regularization is to allow and explicit calculation of the regularized eigenfunctions.

Imposing Dirichlet boundary condition on the field  $\xi$  on a small circle and then making the radius of the circle vanish, reproduces eq.(18) and eq.(19) only for  $1 < \alpha < 2$ . At the same time the field  $h = \frac{\partial}{\partial \bar{z}}\xi$  will violate the Dirichlet boundary condition in the same range of  $\alpha$ , as  $\delta = 2\alpha - 1$  no longer lies in the limits  $0 < \delta \leq 1$ . With Neumann boundary conditions ( $\frac{\partial}{\partial \bar{z}}\xi = 0$ ) we have the same situation in the interval  $\frac{1}{2} < \alpha < 1$ .

Our boundary conditions span the whole range  $\frac{1}{2} < \alpha < 2$ , at the boundary of which both the  $L^2$  character of the eigenfunctions and the procedure for obtaining eq.(11), for  $\delta = \alpha - 1$  or  $\delta = 2\alpha - 1$ , are lost.

For  $0 < \alpha \leq \frac{1}{2}$  with our boundary conditions we obtain  $\delta = \alpha$  and for  $N < \alpha \leq N + 1$  we have  $\delta = \alpha - N$ , thus introducing a non analytic behavior of  $\det' L^\dagger L$  as a function of  $\alpha$ . This is not completely unexpected as a non analytic behavior is already present in  $c_0$  for the Green function of a particle on a cone as a function of the magnetic flux through the tip of the cone [16], which in our case represents the phase change. The fact that the analytic continuation of the eigenfunctions outside the range  $1/2 < \alpha < 2$  are no longer  $L^2$  integrable suggests that an analytic extension of eq.(18) and eq.(19) to the whole range  $0 < \alpha$  requires a definition of the determinant of a transformation on non  $L^2$  functions.

Coming back to eq.(7) and performing a variation of the conformal factor we obtain

$$\begin{aligned} -\delta \ln \frac{\det'(L^\dagger L)}{\det(\Psi_i, \Psi_j)} &= \gamma_E \delta c_0^K + \sum_i \left\{ (\delta \lambda_i - \lambda_i \frac{\delta \alpha_i}{\alpha_i}) [4c_0^K_i - 2c_0^H_i] + \right. \\ &\left. + \text{Finite}_{\epsilon \rightarrow 0} \left[ 4 \frac{\delta \alpha_i}{\alpha_i} \int dx \ln(\alpha_i |x|) K_{\alpha_i}(x, x, \epsilon) - 2 \frac{\delta \alpha_i}{\alpha_i} \int dx \ln(\alpha_i |x|) H_{\alpha_i}(x, x, \epsilon) \right] \right\}. \end{aligned} \quad (20)$$

A differential of this structure [14] can be integrated to give

$$\begin{aligned} \ln \sqrt{\frac{\det'(L^\dagger L)}{\det(\Psi_i, \Psi_j)}} &= \\ &= \frac{26}{12} \left\{ \sum_{i,j \neq i} \frac{(1 - \alpha_i)(1 - \alpha_j)}{\alpha_i} \ln |w_i - w_j| + \lambda_0 \sum_i (\alpha_i - \frac{1}{\alpha_i}) - \sum_i F(\alpha_i) \right\} \end{aligned} \quad (21)$$

where  $F(\alpha)$  is given by a well defined and convergent integral representation.

In the continuum limit, i.e. small angular deficits  $1 - \alpha_i$  and dense set of  $\omega_i$ , the first two terms of (21) go over to the well know continuum formula

$$\frac{26}{96\pi} \left\{ \int dx dy (\sqrt{g}R)_x \frac{1}{\square}(x, y) (\sqrt{g}R)_y - 2(\ln \frac{A}{\bar{A}}) \int dx \sqrt{g}R \right\} \quad (22)$$

as can be easily checked, where  $A$  is the area  $\int dx \sqrt{g}$  and  $\bar{A}$  is the area evaluated for  $\lambda_0 = 0$ . The remainder  $\sum_i F(\alpha_i)$  goes over to a constant topological term.

In addition to having the correct continuum limit eq.(21) has the following appealing features:

- i) It is an exact result giving the value of the *F.P.* determinant related to the conformal gauge-fixing on a two dimensional Regge surface.
- ii) It is invariant under the group  $SL(2, C)$  which acts on  $\omega_i$  and  $\lambda_0$  as

$$\begin{aligned}\omega_i &\rightarrow \omega'_i = \frac{a\omega_i + b}{c\omega_i + d} \\ \lambda_0 &\rightarrow \lambda'_0 = \lambda_0 + \sum_i (\alpha_i - 1) \ln |\omega_i c + d|\end{aligned}\tag{23}$$

and leaves the  $\alpha_i$  unchanged.

iii) While  $\alpha_i > 0$  with  $\sum_i (1 - \alpha_i) = 2$ , the  $\omega_i$  vary without restriction in the complex plane. As pointed out in [12] this is an advantage over the equivalent parameterization of the Regge surface in term of the bone lengths  $l_i$  where one has to keep into account of a large number of triangular inequalities.

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